CONTINUITY & DIFFERENTIABILITY- MODULE 3

RECAP

DIFFERENTIATE THE FOLLOWING

1) $y=x^n \sin x$
2) y =(ax+b)(cx+d)
3) $y = \frac{x^n}{\sin x}$
4) $y = \frac{\sin x}{e^x}$
5) $y = 5x^4+8x^3-7x^2+10x-2$

1) $\frac{dy}{dx} = x^n \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^n)$

$= x^n \cos x + \sin x .\ n x^{n-1}$

3) $\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}(x^n) - x^n \frac{d}{dx}(\sin x)}{\sin^2 x} =$

$\frac{\sin x .\ n x^{n-1} - x^n \cos x}{\sin^2 x}$

5) $\frac{dy}{dx} = 5.4x^3 + 8.\ 3x^2 - 7.2x^1 + 10$

$= 20x^3 + 24x^2 - 14x + 10$

$$\frac{d}{dx}(f(ax+b) = a.f'(ax+b)$$

Find $\frac{dy}{dx}$

1)Y = sin(2x+6)

2) Y = e^{7x-9}

3) Y = log (-5x+1)

4) Y = sec($\frac{1}{2}$x-3)

- 2cos (2x+6)
- 7. e^{7x-9}
- $\frac{1}{-5x+1}$(-5)
- sec($\frac{1}{2}$x-3) tan($\frac{1}{2}$x-3) ($\frac{1}{2}$)

$$\frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)$$

Y= sin^5x	$\frac{dy}{dx}$= $5sin^4x$.cosx
Y= $\sqrt{x^2+5x+7}$	$\frac{dy}{dx} = \frac{1}{2}(x^2+5x+7)^{\frac{-1}{2}}(2x+5)$
Y = $(x^2+7x+tanx)^5$	$\frac{dy}{dx}$= 5 $(x^2+7x+tanx)^4$(2x+7+sec^2x)

$$\frac{d}{dx}[f^n(g(x))] = n f^{n-1}(g(x)).\ f'(g(x)\ g'(x)$$

- $Y = sin^5(3x^2+2)$

$$\frac{dy}{dx} = 5.\ sin^4(3x^2+2).COS\ (3x^2+2)\ .\ 6X$$

PFA

POWER,
FUNCTION,
ANGLE

- $Y = cos^3(e^x)$

$$\frac{dy}{dx} = 3.\ cos^2(e^x).\ (-\ sin(e^x).\ (e^x)$$

$$y = \log\left(x+\sqrt{x^2+1}\right)^2$$

$$y = 2.\log(x+\sqrt{x^2+1})$$

$$\frac{dy}{dx} = 2.\frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{1}{2\sqrt{x^2+1}}x2x\right)$$

$$= 2.\frac{1}{x+\sqrt{x^2+1}}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$$

$$= 2\frac{1}{\sqrt{x^2+1}} = \frac{2}{\sqrt{x^2+1}}$$

- $Y = [\log(x + \sqrt{x^2+1})]^2$

$$\frac{dy}{dx} = 2(\log(x+\sqrt{x^2+1})) \times \frac{1}{x+\sqrt{x^2+1}}(1+\frac{1}{2\sqrt{x^2+1}} \times 2x)$$

$$= 2.(\log(x+\sqrt{x^2+1})\frac{1}{x+\sqrt{x^2+1}}(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}})$$

$$= \frac{2(\log(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}$$

PFA

POWER,

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$$Y=\sqrt{e^{\sin x}} = (e^{\sin x})^{\frac{1}{2}}$$

$$= \frac{1}{2}(e^{\sin x})^{-\frac{1}{2}}.\ e^{\sin x}.\text{COSX}$$

$$= \frac{1}{2}\sqrt{e^{\sin x}}.\text{COSX}$$

Find the derivative of y = $(2x+1)^5(x^3-x+1)^4$

Chain rule
with product r

$$\frac{dy}{dx} = (2x+1)^5\frac{d}{dx}(x^3-x+1)^4 + (x^3-x+1)^4\frac{d}{dx}(2x+1)^5$$

$$= (2x+1)^5 \cdot 4(x^3-x+1)^3\frac{d}{dx}(x^3-x+1) + (x^3-x+1)^4 \cdot 5(2x+1)^4\frac{d}{dx}(2x+1)$$

$$= 4(2x+1)^5(x^3-x+1)^3(3x^2-1) + 5(x^3-x+1)^4(2x+1)^4 \cdot 2$$

$$= 2(2x+1)^4(x^3-x+1)^3[2(2x+1)(3x^2-1) + 5(x^3-x+1)]$$

$$= 2(2x+1)^4(x^3-x+1)^3[(4x+2)(3x^2-1) + 5x^3 - 5x + 1]$$

$$= 2(2x+1)^4(x^3-x+1)^3[12x^3 - 4x + 6x^2 - 2 + 5x^3 - 5x + 5]$$

$$= 2(2x+1)^4(x^3-x+1)^3(17x^3 + 6x^2 - 9x + 3)$$

$$Y = \frac{(2x-5)^4}{(8x^2-5)^3}$$

Chain rule with QUOTIENT

Find the derivative of $y = (2x-5)^4(8x^2-5)^{-3}$

Solution

We need to use the product rule and the chain rule:

$$y' = 4(2x-5)^3(2)(8x^2-5)^{-3} + (2x-5)^4(-3)(8x^2-5)^{-4}(16x)$$

The rest is a bit of algebra, useful if you wanted to solve the equation $y' = 0$:

$$\begin{aligned} y' &= 8(2x-5)^3(8x^2-5)^{-4}\left[(8x^2-5) - 6x(2x-5)\right] \\ &= 8(2x-5)^3(8x^2-5)^{-4}\left(-4x^2+30x-5\right) \\ &= -8(2x-5)^3(8x^2-5)^{-4}\left(4x^2-30x+5\right) \end{aligned}$$

WHAT IS CHAIN RULE

y = f [g(x)]

let t = g(x) then y = f(t) : $\frac{dy}{dt} = f'(t)$ and $\frac{dt}{dx} = g'(x)$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = f'(t) \cdot g'(x) = f'(g(x)) \cdot g'(x)$$

y = f [g(h(x))]

let t=h(x) , and then y = f(g(t)) : let u = g(t) then y = f(u)

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx} = f'(u) \cdot g'(t) \cdot h'(x) = f'(g(h(x))). g'(h(x)). h'(x)$$

EXAMPLE:Y = $sin^5(3x^2 + 2)$

t = $3x^2 + 2$; u = sin t : y = u^5

$\frac{dy}{dx} = 5. sin^4(3x^2 + 2)$.COS $(3x^2 + 2)$. 6X

HOME WORK

Find the derivative

- 1) $\log(x^2).\sin(\sqrt{x})$
- 2) $\log(\log x).\tan x$
- 3) $\sin^3 x.e^{\sec x}$
- 4) $\sin^2(\sqrt{x})$
- 5) $e^{\sin x}(5+\tan x)$